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SEMINAR
Report

Black Scholes and Implied Volatility

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ABSTRACT

The aim of this project is to present the complexity of relations that exists between the price of option calculated on the basis of Black-Scholes formula and the volatility of underlying instrument which is one of the formula's inputs. The interactions have been examined using an application created in VBA Excel. The assumptions, calculations and results of the analysis are described in the following parts of report.

The Excel application has been applied to analyze European option on OMXS30 index. The calculations were performed for both put and call options and their prices on 28 September 2007. They include:

- Calculating the historical volatility on the entire stock data.
- Calculating the historical volatility for each life-time of the options.
- Calculating the implicit volatility the options with Black-Scholes
- Comparison of the market price with the theoretical price resulting from the historical volatility

PART 1

Introduction

1. Black-Scholes formula for European options

A European Option are options which are only exercisable on the expiry date of the option and are valued using the Black Scholes option pricing formula. There are five inputs to the classic Black Scholes model: spot price, strike price, expiration time, interest rate, and volatility. As such European options are typically the least complicated options to value The dividend or yield on the underlying asset can also be an input on some extensions of the model. However the option price considered in the project does not depend on dividend since it is an index option.

The Black-Scholes formula for European option is the following:

$$P_{call} = S \cdot N(d_1) - K \cdot e^{-rT} \cdot N(d_2)$$

$$P_{put} = K \cdot e^{-rT} \cdot N(-d_2) - S \cdot N(-d_1)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + rT}{\sigma\sqrt{T}} + \frac{1}{2}\sigma\sqrt{T} \qquad d_2 = d_1 - \sigma\sqrt{T}$$

where:

S – price of underlying instrument

K – strike price

T – expiration time

σ – volatility

r – risk-free interest rate

$N(d_1)$, $N(d_2)$ – value of normal distribution for argument d_1 or d_2 respectively

2. Underlying index

2.1 Basic information

The following analysis is conducted for put and call options on index OMX Stockholm 30. It is the Stockholm Stock Exchange's leading share index. The index consists of the 30 most actively traded stocks on the Stockholm Stock Exchange. The limited number of constituents guarantees that all the underlying shares of the index have excellent liquidity, which results in an index that is highly suitable as underlying for derivatives products. In addition OMXS30 is also used for structured products, e.g. warrants, index bonds, exchange traded funds such as XACT OMX™ and other non-standardized derivatives products. The composition of the OMXS30 index is revised twice a year. The OMXS30 Index is a market weighted price index. The base date for the OMX Stockholm 30 Index is September 30, 1986, with a base value of 125.[1]

2.2 Collected data

The historical daily data were downloaded from OMX website
http://www.omxgroup.com/nordicexchange/priceinformation/historical_prices/

The option on OMXS30 that is analyzed expires on 28th December 2007.

2.3 Time horizon

The analysis is conducted with respect to date 28-09-2007. However the time horizon of collected data reaches as far as 4 years behind. The starting point is therefore the 29-09-2003.

The closing prices of this index within this time period are presented on the Figure 1 below.

[1] SOURCE: OMX web site
http://www.omxgroup.com/nordicexchange/Products/indexes/OMX_index_family/OMXS_Local_Index/

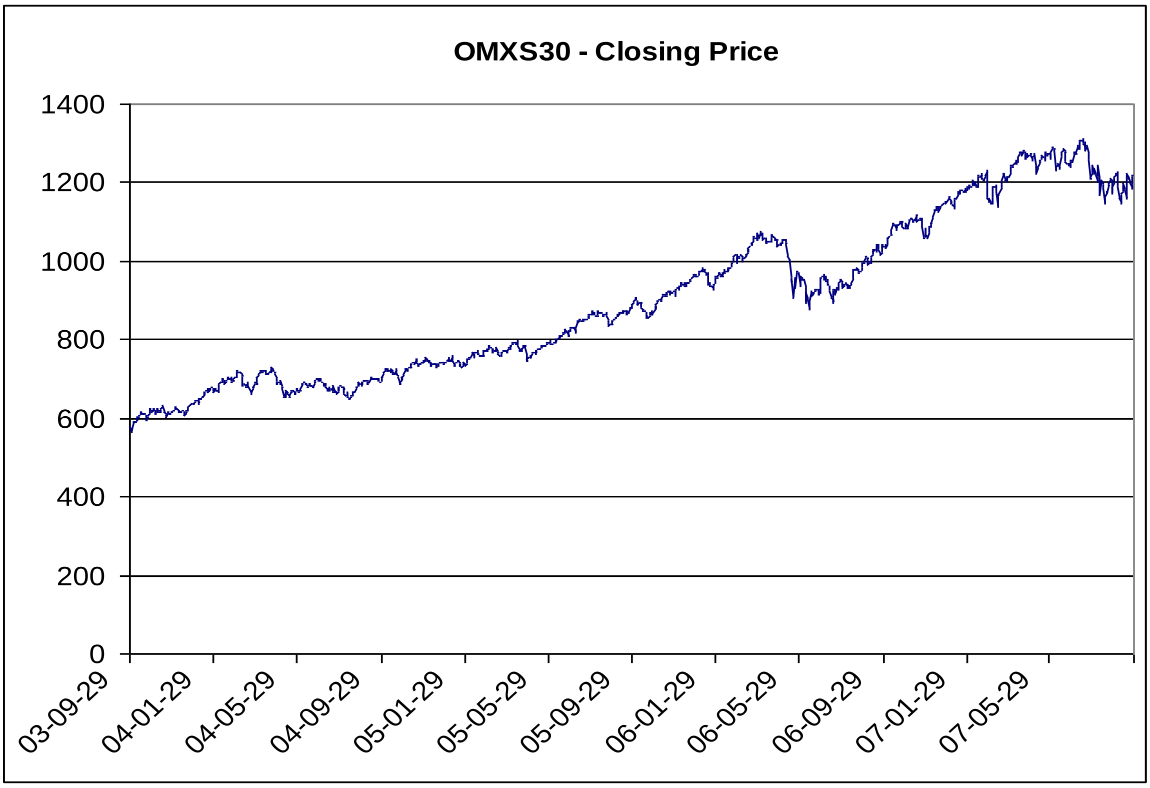


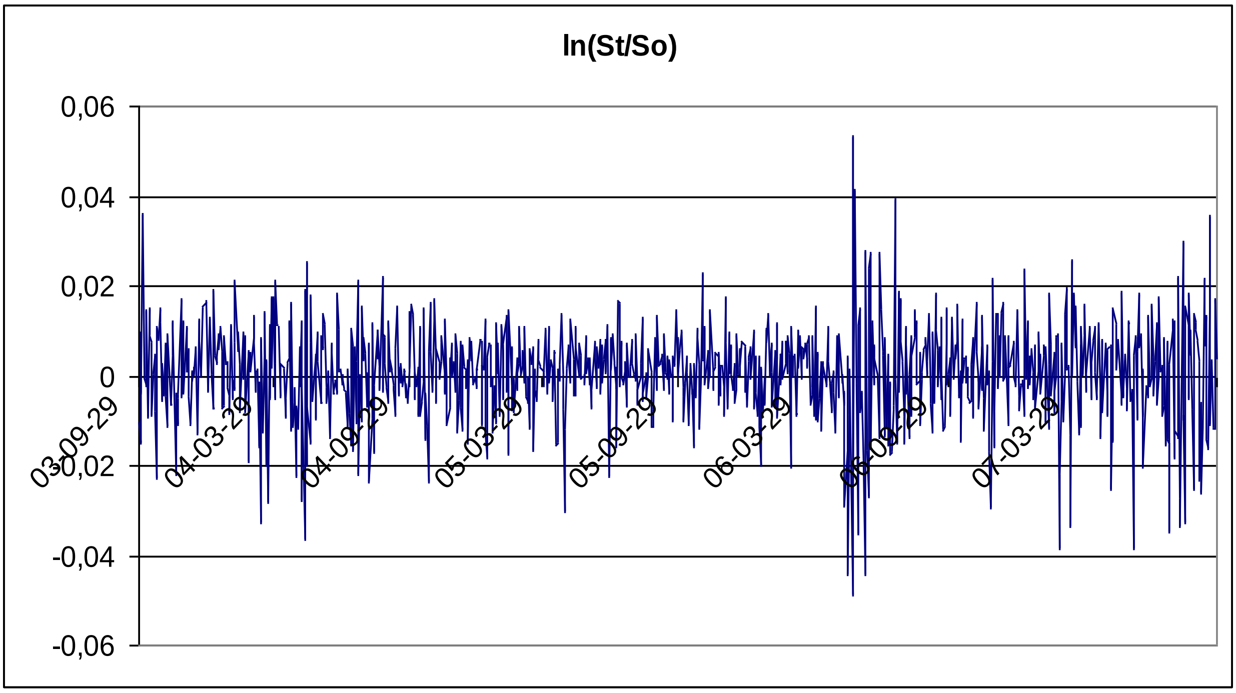
Figure 1 OMXS30 Closing Price [29-09-2003 - 28-09-2007]

PART 2

Historical Volatility

For the purpose of our project we had to calculate a historical volatility of the prices of the OMXS30 index. All calculations are based on data from 2003.09.29 to 2007.09.28.

Below we can see the graph of the logarithmic rates of return from OMXS30.



Those different rates make us interested about the volatility of the OMXS30's prices.

The formula for historical volatility is $s\sqrt{d}$, where d is the number of tradings days in a year (250) and s is standard deviation given with formula above:

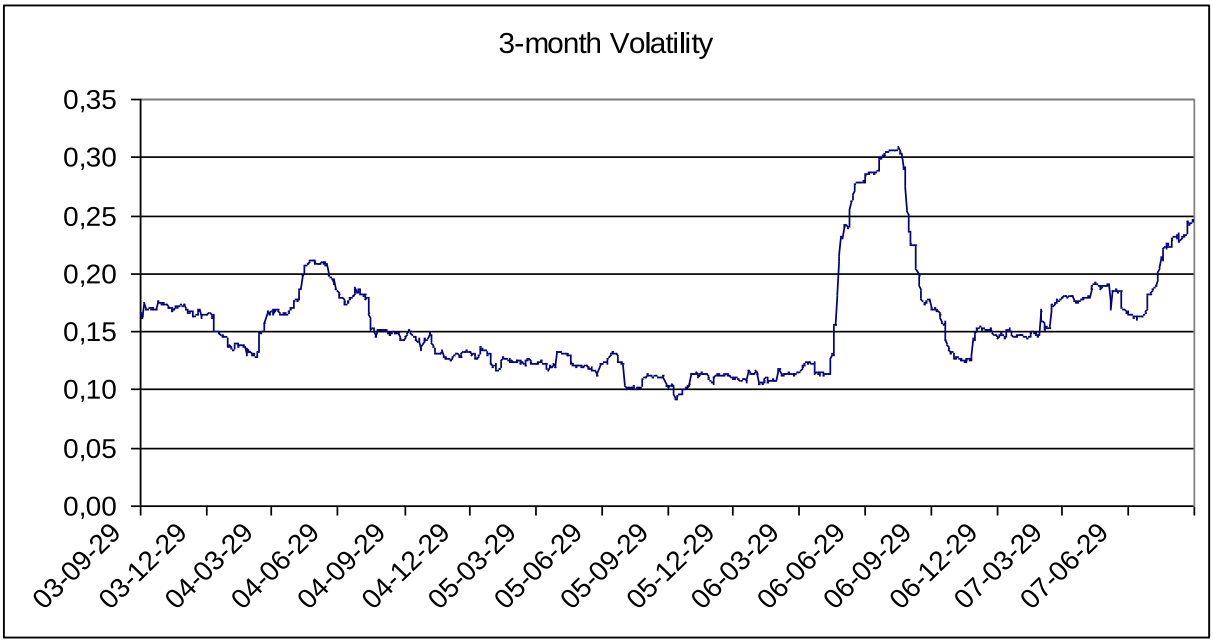
$$s = \sqrt{\frac{1}{1-n}\sum_{i=1}^{n} u_i^{2} - \frac{1}{n \cdot (n-1)}\left(\sum_{i=1}^{n} u_i\right)^2}$$

n – numbers of analize period

u_i – $\ln(a_i/a_{i-1})$

a_i – price from period i

First of all we calculated the 3-month volatility from the given period. Here is the graph that depicts how the volatility changes through the time.



But for us the crucial information is the volatility from last three months, which will be used in Black-Scholes formula for calculating price of an option at the 28.09.2007.

To sum up for pricing the option we need tree kinds of volatilities:

- three month volatility
- average from whole calculates three month volatilities
- volatility from whole analysed period, that is 4 years

Here are the results of our calculations:

3M volatility	averaage 3M volatility	4Y volatility
24,6309%	15,6949%	16,70699%

PART 3

Implied Volatility

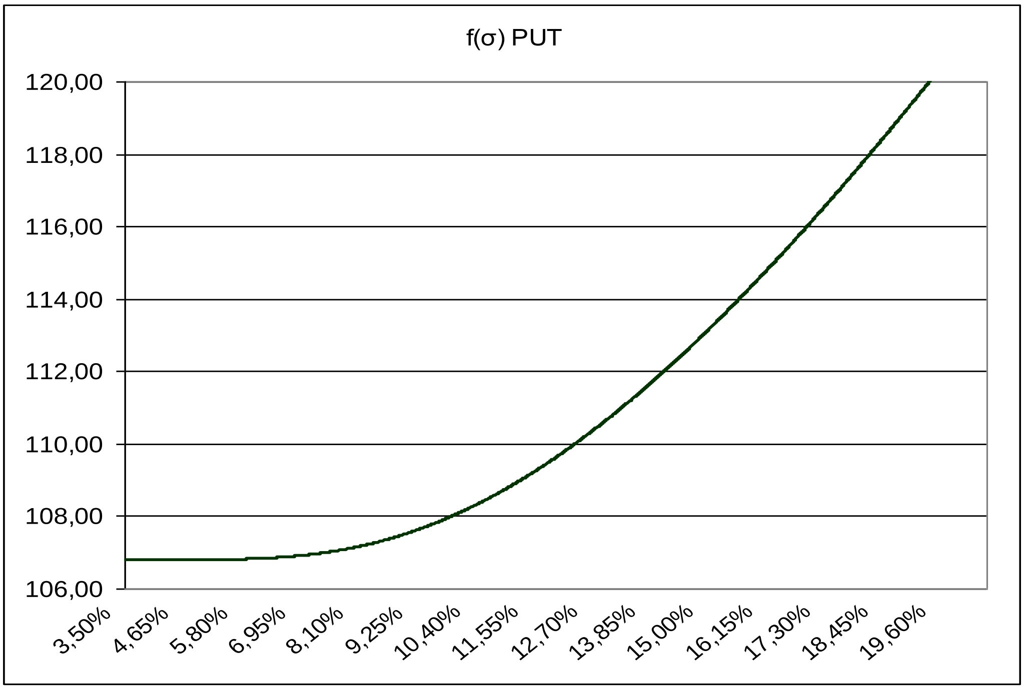
Volatility is the only input in Black-Scholes formula that cannot be observed directly but needs to be calculated each time. The easiest way to obtain information concerning the level of fluctuations of prices is to calculate so called historical volatility basing on historical data. This procedure was conducted and presented in previous part.

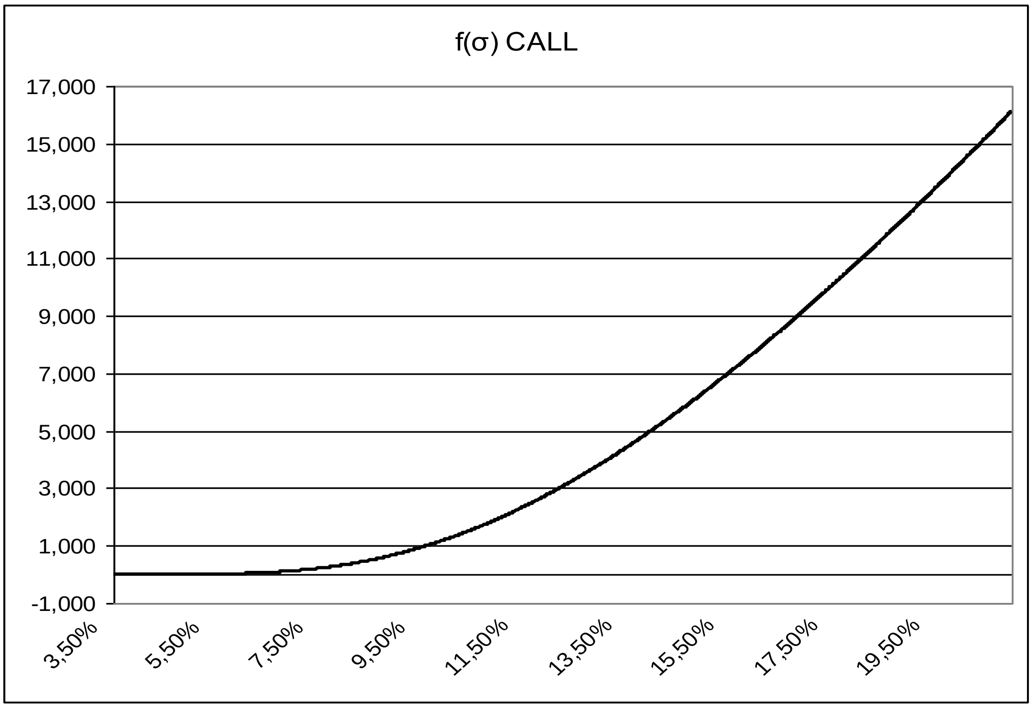
However it turns out that in practice, traders usually work with what are known as implied volatilities. Theses are the volatilities implied by option prices observed in the market.[2] Finding implied volatility of the index is used to verify the market's opinion about the volatility of this index (or stock).

In our project we created a VBA application which calculates the implicit volatility for Black-Scholes pricing formula. Two separate sheets are provided with macros that calculate implied volatility for put and call options respectively.

The following figures show the relation between the level of volatility of underlying instrument and the price of option (put or call) resulting from Black-Scholes formula. As we can see both relations are positive and for both types of options their prices increases much faster for high level volatilities (over 10%). The difference between the options appears when we compare the absolute prices of both options for the same level of volatility. We observe that for analyzed option, for which strike price reaches much higher level than price of underlying index 1340 compared to 1221,44), put option is much more valuable than call option. That obviously results from a natural respond of the market and its participants who in this case prefer purchasing put options on OMXS30.

[2] „Options, Futures and Other Derivatives” Fifth edition; by John C. Hull p. 250





Moreover the conclusions that can be derived from the calculations of implied volatility are the following:

- the value of implicit volatility changes when we change the strike price (other things remaining unchanged). The relation between those variables is shown on the Figure 2 below. The volatilities were calculated for the put option on OMXS30 at its given market price on the 28th September 2007 (which was 112SEK), given expiration time (3 months), value of underlying index (1221,54SEK) and risk free rate (3,5%).

We observe that there exist a negative relation which means that the level of implied volatility drops as the strike price increases.

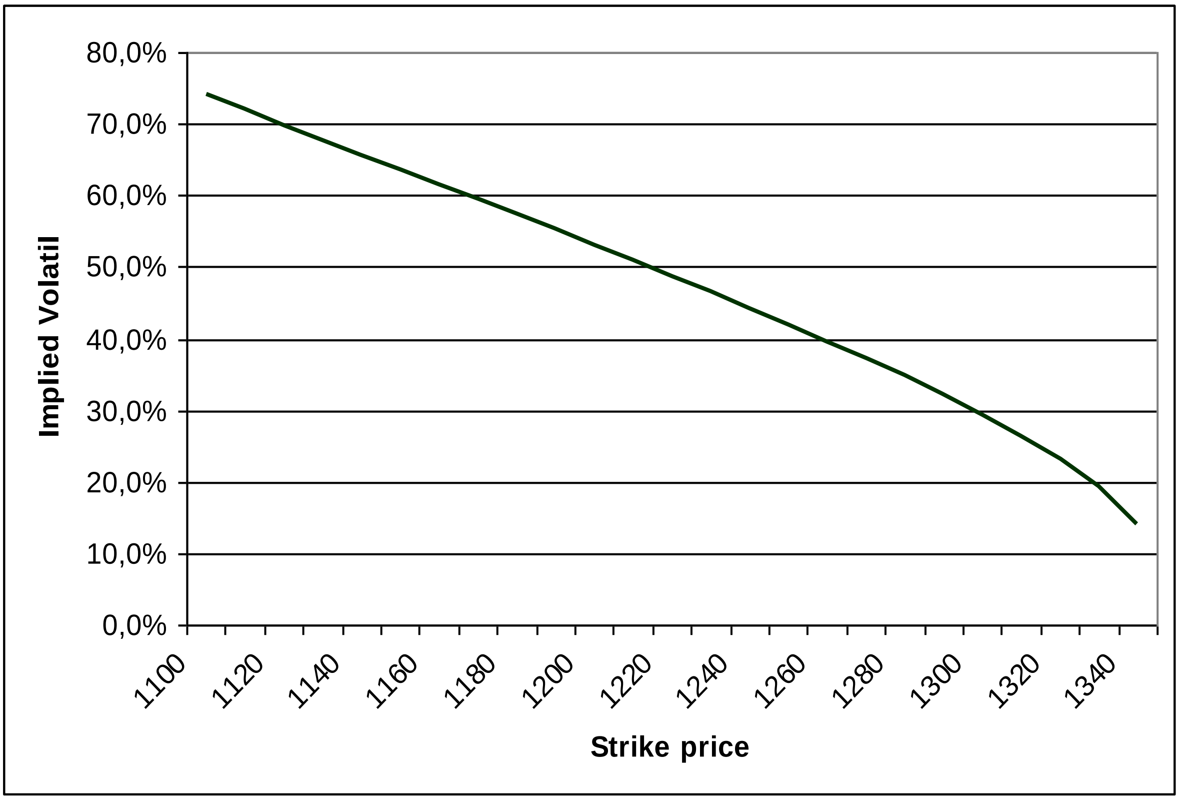


Figure 2 Implied volatility with respect to strike price

○ the value of implicit volatility changes when we change the time to maturity (other things remain unchanged). The following Figure 3 presents the complexity of this relation.

As we see the curvature of volatility calculated by the VBA application is humped. At first it increases sharply but as the expiration reaches the level of around 0,5 year the volatility starts descending. However this trend reverts again after the expiration of 1,4 year.

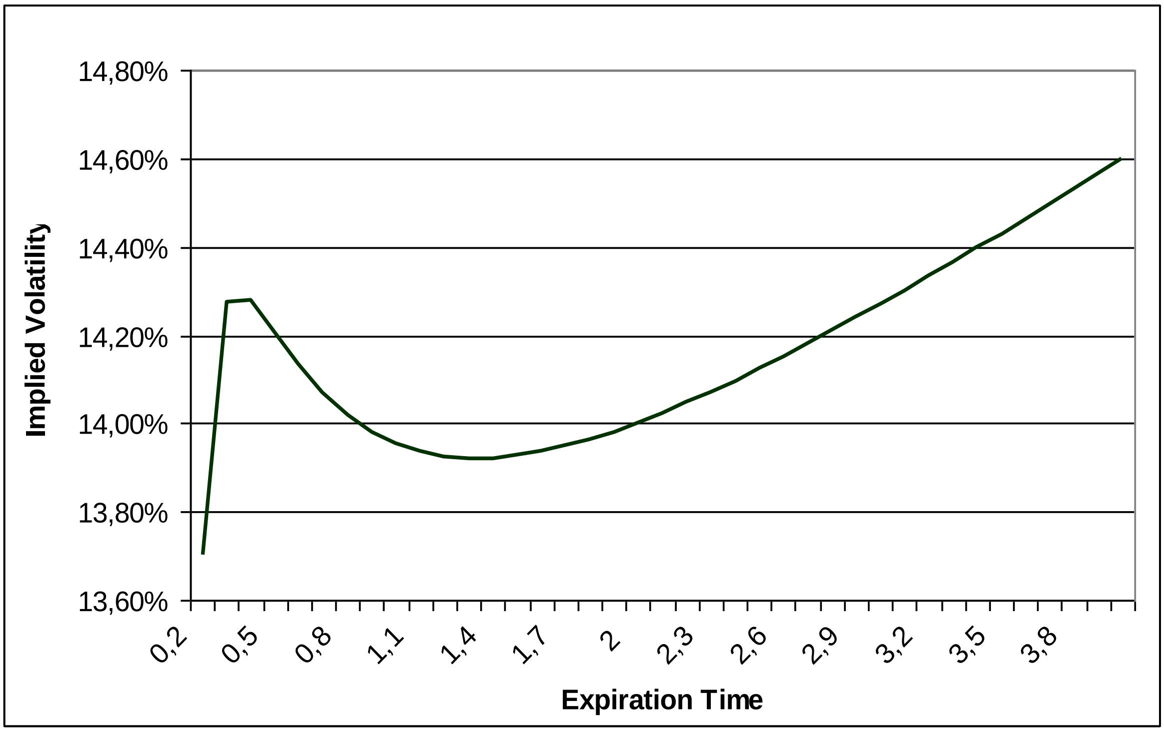


Figure 3 Implied volatility with respect to expiration time

Comparing implied volatility with historical one:

Volatility	3m	average 3m	4y	Implied volatility
	24,63087%	15,69488%	16,70699%	14,1482%(Put) 21,2017%(Call)

The aim of this project is to present the relation between different kinds of volatility. Using the VBA application we calculated the level of implied volatility which reflects how much volatility the market currently assumes within the Black-Scholes framework.

As we can see the value that implied volatility takes differs from all 3 kinds of historical one. However there are some further conclusions that we can find. To find out which method of calculating historical volatility we should use to price the option on the particular day, we can compare them with implied one (which is the desired one). The table above implies that the historical volatility calculated on basis of average 3-month volatilities seems to reflect the implied one the best. That suggests that 3-month average volatility is the most appropriate in pricing the option. However we should also remember that implied volatility itself does depend on the expiration time and strike price that the option has. Therefore the conclusions may vary depending on analyzed instrument and its features.

We can observe also difference between implied volatility for put option (14,1482%) and for call one (21,2017%). The explanation of this situation is that having the same strike price, expiration date, free interest rate etc., there is different market price for put and call option. This is the main reason of such different calculated implied volatilities. However for both, call and put option as well, is true that the higher market price is then the higher implied volatility we would get.

PART 4

Price Comparison

Basing on volatilities, which we have calculated before, now we can calculate the theoretical price of the option.

Option theoretical price (28-09-2007) VBA			
Volatility	3M	4Y	average 3M
	24,63087%	16,70699%	15,69490%
CALL	23,13904524	8,805654842	7,301579655
PUT	129,9251928	115,5918024	114,0877272

We can observe that the higher volatility then the higher price of the option we get, either for call or put option. The financial explanation is that high volatility is connected with high risk, so the price of derivative instrument is also high.

But what is most interesting for us is how those value of different volatilities can be used as a prediction for calculating the actual price of instrument.

To get some overview on this problem lets compare a market price of an option with prices we have calculated. The market data is:

Market price	Bid	Ask	Closing
CALL	15	18	16,5
PUT	109,5	114,5	112

Firstly lets analyze the call option price. The market price of the option on 2007.09.28 was 16,5 and if we want to find the closest calculated price then the one based on three month volatility seems to be the most appropriate one. However all those prices calculated on the basis of historical volatility does not explain in right way the actual price. We can only assume that volatility from last closest period gives the best results.

Now lets consider how this relation in the put option is. Here the market price is explained in the best way by price, which was calculated using average three month volatility (market price 112, calculated price 114). We can observe big difference in prices comparing the market one and the one based on three month volatility. We can explain it by substantial increase in volatility in last 2 weeks, whereas there are also other factors influencing the market price, not only short term one.

The general conclusion is that if we want to make some simulation about prices of option, then to get the most appropriate result we should use historical volatility taken from the closest period to our day, on which we are going to do the forecast. What is more we can suggest that the average volatility from several short periods depicts the real volatility in the best way, because then we can get rid of some single disturbtions that may appears from time to time and our calculations become more accurate.

Another conclusion is that volatility from whole analyzed period (for example 4 years) is not a good source to make any forecast about the prices. This kind of volatility would not explain how the situation on the market looks like now and prices would not change today in the same way as it was 4 years ago.